8.5 Higher Order Derivatives

Functions: f, y, u, v
Independent variable: x
Natural number: n

811. Second derivative

$$f'' = (f')' = \left(\frac{dy}{dx}\right)' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

812. Higher-Order derivative

$$f^{(n)} = \frac{d^n y}{dx^n} = y^{(n)} = \left(f^{(n-1)}\right)'$$

813. $(u+v)^{(n)} = u^{(n)} + v^{(n)}$

814. $(u-v)^{(n)} = u^{(n)} - v^{(n)}$

815. Leibnitz's Formulas

$$(uv)'' = u''v + 2u'v' + uv''$$

$$(uv)''' = u'''v + 3u''v' + 3u'v'' + uv'''$$

$$(uv)^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{1\cdot 2}u^{(n-2)}v'' + \ldots + uv^{(n)}$$

816. $\left(x^m\right)^{(n)} = \frac{m!}{(m-n)!}x^{m-n}$

817. $\left(x^n\right)^{(n)} = n!$

818. $(\log_a x)^{(n)} = \frac{(-1)^{n-1}(n-1)!}{x^n \ln a}$

819. $(\ln x)^{(n)} = \dfrac{(-1)^{n-1}(n-1)!}{x^n}$

820. $\left(a^x\right)^{(n)} = a^x \ln^n a$

821. $\left(e^x\right)^{(n)} = e^x$

822. $\left(a^{mx}\right)^{(n)} = m^n a^{mx} \ln^n a$

823. $(\sin x)^{(n)} = \sin\left(x + \dfrac{n\pi}{2}\right)$

824. $(\cos x)^{(n)} = \cos\left(x + \dfrac{n\pi}{2}\right)$